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Intersecting D -Branes in Ten and Six Dimensions

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ABSTRACT

We show how, via T -duality, intersecting D -Brane configurations in ten (six) dimensions can be obtained from the elementary D -Brane configurations by embedding a Type IIB D -Brane into a Type IIB Nine-Brane (Five-Brane) and give a classification of such configurations. We show that only a very specific subclass of these configurations can be realized as (supersymmetric) solutions to the equations of motion of IIA/IIB supergravity.

Whereas the elementary D -brane solutions in $d = 10$ are characterized by a single harmonic function, those in $d = 6$ contain two independent harmonic functions and may be viewed as the intersection of two $d = 10$ elementary D -branes. Using string/string/string triality in six dimensions we show that the heterotic version of the elementary $d = 6$ D -Brane solutions correspond in ten dimensions to intersecting Neveu-Schwarz/Neveu-Schwarz (NS/NS) strings or five-branes and their T -duals. We comment on the implications of our results in other than ten and six dimensions.

1 Introduction

Recent developments have shown that an important role in understanding nonperturbative string theory is played by p -Brane solutions of the string effective action whose charge is carried by a single Ramond/Ramond (RR) gauge field. These solutions correspond to open string states whose endpoints are constrained to live on a $p + 1$ -dimensional worldvolume and are known as Dirichlet p -Branes or simply D -Branes [1] (for a review, see [2]). More precisely, a Dirichlet p -Brane in ten dimensions is an open string state which satisfies Dirichlet boundary conditions for the $9 - p$ transverse directions and Neumann boundary conditions for the $p + 1$ worldvolume directions. Since under T -duality¹ Dirichlet and Neumann boundary conditions are interchanged it follows that all Dirichlet p -Branes ($p = 0, \dots, 9$) are T -dual versions of each other [2]. It is natural that this T -duality is also realised on the p -Brane solutions and indeed this has shown to be the case for all values of p with $0 \leq p \leq 9$ [4].

The elementary Dirichlet p -Brane solutions in ten dimensions are characterized by a single function H_p that depends on the $9 - p$ transverse coordinates and is harmonic with respect to these variables. The metric for all values of p ($0 \leq p \leq 9$) is given by

$$ds^2 = H_p^{-1/2} ds_{p+1}^2 + H_p^{1/2} ds_{9-p}^2. \quad (1)$$

For even (odd) p this metric corresponds, together with certain expressions for the dilaton and the RR gauge field, to a solution of IIA (IIB) supergravity. The only nontrivial T -duality rule involving the metric is given by

$$\tilde{g}_{xx} = 1/g_{xx}. \quad (2)$$

where x labels the isometry direction over which the duality is performed. Clearly, under this duality transformation the metric of a Dirichlet p -Brane becomes that of a $p+1$ -Brane if the duality is performed over one of the transverse directions of the p -Brane. In other words, one of the transverse directions of the p -Brane has become a worldvolume direction of the $p + 1$ -Brane. We assume here that the harmonic function H_p is independent of the particular transverse direction which is dualized or, alternatively, that we consider a periodic array of p -Brane solutions. We therefore may write

$$\tilde{H}_p = H_{p+1}. \quad (3)$$

Conversely, a $p + 1$ -Brane becomes a p -Brane if the duality is done over one of the worldvolume directions of the $p + 1$ -Brane. In this case, in order to establish a duality between the two solutions, we assume that after duality the harmonic function H_{p+1} becomes dependent on this particular worldvolume direction. It is in this sense that we may write

$$\tilde{H}_{p+1} = H_p. \quad (4)$$

¹For a review, see [3].

Strictly speaking the T duality rules can only be applied as solution generating transformations to construct $p + 1$ -Brane solutions out of p -Brane solutions and not the other way around. Therefore the 0-Brane leads, via T -duality, to all other D -Brane solutions. *A priori* it is not guaranteed that one can apply T -duality also as a solution-generating transformation to construct p -Brane solutions out of $p + 1$ -Brane solutions. In the case of the D -brane solutions the T -duality does generate new dual solutions but, as we will see later, this is not true for other configurations. For the general case one must check by hand whether the dual metric indeed is a new solution. In this paper we will use T -duality only as a tool to construct a natural Ansatz for a class of solutions.

A particularly interesting case, that will play an important role later, is the nine-Brane solution which has no transverse directions and whose worldvolume is ten-dimensional Minkowski spacetime. Therefore, all Dirichlet p -Branes are, via T -duality in the $9 - p$ transverse directions, T -dual to flat spacetime. Conversely, out of flat spacetime we can construct all other D -Brane solutions via T -duality. This can be done in the following way. First we write the nine-Brane solution as

$$ds^2 = H_9^{-1/2} ds_{10}^2, \quad (5)$$

where H_9 is a constant (which is related to the spacetime volume). To obtain the 8-Brane solution, we dualize in one of the worldvolume directions, say x^9 , and assume that after duality H_9 becomes dependent on x^9 , i.e. $\tilde{H}_9 = H_8$. One thus obtains the 8-Brane solution. Similarly, one can obtain all other D -Brane solutions. We conclude that not only does the 0-Brane, via T -duality, leads to all other Dirichlet p -Branes with $0 < p \leq 9$ but also does the nine-Brane lead, via T -duality, to all remaining Dirichlet p -Branes with $0 \leq p < 9$.

It is natural to consider bound states of (orthogonal) intersections of D -Branes. Such multiple D -Brane configurations have been considered in [5, 2]². It turns out that, whereas the elementary D -Brane solutions are described by a single harmonic function, the solution corresponding to q intersecting D -Branes contains q independent harmonic functions. Thus, these solutions may be considered as compositions of the elementary solutions. For simplicity, we will first restrict ourselves to solutions with two independent harmonic functions. Several examples of intersecting solutions in ten dimensions (not necessarily D -Branes but also NS/NS solutions) with more than one harmonic function have been given in the literature. The configuration given in [8] describes two NS/NS 5-Branes intersecting over a string. The solution given in eq. (2.5) of [9] describes 3 NS/NS 5-Branes intersecting over a string, while the solution given in eq. (5.2) of the same reference describes the intersection of a fundamental string with two NS/NS 5-Branes. The chiral null model of [10] describes the composition of a NS/NS string and its T -dual while the solution given in [11, 12] describes a fundamental string and its T -dual lying within a NS/NS 5-Brane and its T -dual. Finally, the solution of [13] describes a Dirichlet 1-Brane inside a Dirichlet 5-Brane.

Recently, intersecting p -Brane solutions have also been considered in eleven dimensions

² Generalizations of D -Branes where an open p -Brane ends on a q -Brane have been discussed in [6, 7].

in which case they are referred to as intersecting M -Branes. More explicitly, it has been shown in [14] that the solutions of [15] that break more than one half of the eleven-dimensional supersymmetry can be considered as intersections of the elementary 2-Branes [16] and 5-Branes [15]. This analysis has been extended in [17] where intersections with independent harmonic functions for each elementary constituent has been considered. The discussion of [17] was also applied to ten dimensions where several examples of solutions describing intersecting D -Branes were given. For all these solutions, the general rule seems to be that the intersection of q solutions preserves at most $(1/2)^q$ supersymmetry.

It is the purpose of this work to give a systematic and unified treatment of solutions describing intersecting Dirichlet p -Branes in ten and six dimensions. For this purpose we first consider the elementary D -Brane solutions in general dimensions³. We define a solution to be an elementary Dirichlet p -Brane if (i) it is charged with respect to one or more RR gauge fields, (ii) the metric contains a single function H_p which is harmonic with respect to the transverse variables and has the property that the worldvolume and transverse directions are multiplied with *opposite* powers of this function and (iii) the solution has vanishing scalar except for a possible dilaton. From the requirement (ii) it follows that the metric of all elementary D -Branes are related via T -duality, a property which we expect from an open string state with mixed Dirichlet/Neumann boundary conditions. The requirement (iii) ensures that the solutions are connected to flat space time with constant scalars.

To determine the metric of an elementary D -Brane solution it is convenient to first consider those D -Branes that have vanishing dilaton. The other D -Brane solutions are then obtained via T -duality. It is not too difficult to see, using the results of [20], that such solutions only exist for $p = (D - 4)/2$ and that the metric of such solutions is given by

$$ds^2 = H_p^{-4/(D-2)} ds_{p+1}^2 + H_p^{4/(D-2)} ds_{D-p-1}^2. \quad (6)$$

For $d = 10$ this reproduces the metric of (1). Instead, we see that for $d = 6$ the elementary D -Brane solutions are given by

$$ds^2 = H_p^{-1} ds_{p+1}^2 + H_p ds_{5-p}^2. \quad (7)$$

We conclude that the elementary D -Branes in six dimensions cannot be obtained from a dimensional reduction of elementary D -Branes in ten dimensions since the two solutions have different powers of harmonic functions in their respective metrics⁴. In this paper we will show, following a suggestion of [14, 17], how all the elementary D -Brane solutions in six dimensions have a higher-dimensional interpretation in terms of intersecting D -Branes in ten dimensions. Furthermore we will construct, by using string/string/string triality [21, 22], the heterotic analogue of the six-dimensional D -Branes and show that their

³ Since the zero-modes of D -Branes are described by a worldvolume vector multiplet we should restrict ourselves to those dimensions in which there is a Bose-Fermi matching on the worldvolume using vector multiplets. This restricts ourselves to $d = 3, 4, 6, 10$ dimensions [19]. Note that only in $d = 6, 10$ we can distinguish between IIA and IIB theories.

⁴The reduction of the elementary $d = 10$ D -Branes to six dimensions lead to solutions with extra nonvanishing scalars since the metric in the compactified directions is nontrivial.

ten-dimensional origin is given by ten-dimensional intersecting Neveu-Schwarz/Neveu-Schwarz (NS/NS) strings or five-Branes and their T -duals.

The organization of this paper is as follows. In section 2 we will first classify all possible intersecting configurations consisting of two elementary D -Branes in ten dimensions and briefly discuss the extension to more than two intersecting D -Branes. After that we will show that only a specific subclass of these configurations can be (supersymmetric) solutions to the equations of motion of IIA/IIB supergravity. In section 3 we will use these results to explain the ten-dimensional origin of the D -Brane solutions in six dimensions. Furthermore we will construct the heterotic analogue of the $d = 6$ D -Branes and explain their ten-dimensional origin. Finally, in the last section we comment on the implications of our results in other than ten and six dimensions.

2 D -Branes and their intersections in $d = 10$

In this section we discuss the intersection of two D -Branes in ten dimensions. To construct configurations which describe intersections, a crucial role is played by T -duality. Let us first explain, via an example, the main idea of this construction. After that we will give a more systematic treatment. Consider an elementary Dirichlet p -Brane with $1 \leq p \leq 7$ odd and metric given in (1). We embed this D -Brane in flat spacetime, i.e. a nine-Brane, and write

$$ds^2 = \frac{1}{\sqrt{H_p H_9}} ds_{p+1}^2 - \sqrt{\frac{H_p}{H_9}} ds_{9-p}^2, \quad (8)$$

where H_9 is the (constant) harmonic of the nine-Brane. In the table at the end we have indicated this embedding by the \parallel symbol, e.g. we write $1\parallel 9, 3\parallel 9$, etc. We next may perform a T -duality transformation in two different ways. The first possibility is that we perform the T -duality over one of the transverse directions of the p -Brane, say x^9 , which from the nine-Brane point of view is a worldvolume direction. In this way we obtain the configuration⁵

$$ds^2 = \frac{1}{\sqrt{H_{p+1} H_8}} ds_{p+1}^2 - \sqrt{\frac{H_{p+1}}{H_8}} ds_{8-p}^2 - \sqrt{\frac{H_8}{H_{p+1}}} (dx^9)^2, \quad (9)$$

where we have used that $\tilde{H}_p = H_{p+1}$ and $\tilde{H}_9 = H_8$. We thus end up with a configuration describing the intersection of a $(p+1)$ -Brane with an 8-Brane via a p -Brane. In the table these new configurations are indicated one step to the right with respect to the original configuration, e.g.

$$1\parallel 9 \rightarrow 2\perp 8. \quad (10)$$

⁵The expressions for the gauge fields and dilaton can easily be obtained by applying the Type II T -duality rules of [23]. They will be given below for the general case.

The second possibility is that we perform the T -duality over one of the worldvolume directions of the p -Brane, say x^1 . This leads to the configuration

$$ds^2 = \frac{1}{\sqrt{H_{p-1}H_8}}ds_p^2 - \sqrt{\frac{H_{p-1}}{H_8}}ds_{9-p}^2 - \sqrt{H_{p-1}H_8}(dx^1)^2, \quad (11)$$

where we have used that $\tilde{H}_p = H_{p-1}$ and $\tilde{H}_9 = H_8$. This configuration describes a $(p-1)$ -Brane embedded into an 8-Brane. In the table these new configurations are indicated one step above the original configuration, e.g.

$$1 \parallel 9 \rightarrow 0 \parallel 8. \quad (12)$$

Clearly, from the configurations (9) and (11) one can obtain further intersecting configurations by performing T -duality over different directions. This leads us to consider the following class of metrics with two independent harmonic functions H_{p+r}, H_{p+s} describing the intersection of a $(p+r)$ -Brane with a $(p+s)$ -Brane via a p -Brane:

$$ds^2 = \frac{1}{\sqrt{H_{p+r}H_{p+s}}}ds_{p+1}^2 - \sqrt{\frac{H_{p+r}}{H_{p+s}}}ds_s^2 - \sqrt{\frac{H_{p+s}}{H_{p+r}}}ds_r^2 - \sqrt{H_{p+r}H_{p+s}}ds_{9-p-r-s}^2. \quad (13)$$

The dilaton is given by

$$e^{-2\phi} = (H_{p+r})^{\frac{p+r-3}{2}}(H_{p+s})^{\frac{p+s-3}{2}}. \quad (14)$$

Using the terminology of [14, 18] we call the first $p+1$ coordinates the worldvolume coordinates of the intersection. The next s and r directions are the “relative transverse” directions with respect to the $p+r$ -Brane and $p+s$ -Brane, respectively. Finally, the last $9-p-r-s$ are the “overall transverse” directions. We recover the metric of the elementary Dirichlet $(p+r)$ and $(p+s)$ -Brane by putting H_{p+s} and H_{p+r} equal to one, respectively. We will denote the configuration given in (13) as $(p+r) \times (p+s)$. Note that if r or s is equal to zero the configuration describes the embedding of one D -Brane into another one. We will indicate this case as $p \parallel (p+s)$. On the other hand, if both r and s are different from zero, the configuration (13) describes two D -Branes which are orthogonally intersecting over a p -Brane. We will indicate this situation as $(p+r) \perp (p+s)$.

The labels p, r and s in the general configuration (13) have to fulfill certain conditions: first of all $p+r+s \leq 9$ for the obvious reason that we only have 9 spatial dimensions to fill. Further we only want to combine objects which come from the same theory (IIA or IIB), so $r+s$ has to be an even number $2n$, where n labels different classes of configurations, as indicated in the table.

In general the configuration $(p+r) \times (p+s)$ transforms under T -duality as follows. One possibility is that

$$(p+r) \times (p+s) \rightarrow (p+(r \pm 1)) \times (p+(s \mp 1)), \quad (15)$$

if under the duality a relative transverse direction of one object becomes a relative transverse direction of the other object. In this case we move horizontally in the table. The second possibility is that we have

$$(p+r) \times (p+s) \rightarrow ((p \pm 1) + r) \times ((p \pm 1) + s), \quad (16)$$

if under duality an overall transverse direction becomes an intersecting one or vice versa.

In either case (15) or (16) the number $r + s = 2n$ remains constant, so that n can be used to label the four different classes given in the table. Within each class we may move via T -duality in the way described above. To go from one class to another we first rewrite the $(p \parallel 9)$ element of the class as an elementary D -Brane p . Next we transform p into q under T -duality and write q as $q \parallel 9$ which is the element of a different class. There is one subtlety here. It only makes sense to embed a p -Brane into a 9-Brane for $(p = \text{odd})$ -Branes. The reason is that one should view the nine-Brane as a IIB solution since under T -duality it is transformed into a IIA solution (the 8-Brane). This implies that it can only intersect with another IIB solution.

In conclusion, we can summarize all possible intersections of two D -Branes in 10 dimensions as given in the table: the blocs at the bottom are the elementary D -Branes, and on each $(p = \text{odd})$ -solutions we can build a tower of intersecting D -Brane configurations, labeled by the integer n . Within each tower we can move from one configuration to the horizontally and vertically neighbouring ones via the different possible T -dualities: horizontal moves, in which p remains constant correspond to dualizing relative transverse coordinates, while vertical moves keep r and s constant and correspond to dualizing worldvolume directions into overall transverse ones or vice versa.

The explicit expressions for the RR fields follow from the T -duality (for more details see [4]). Alternatively, their expression can easily be obtained by the requirement that, if one of the harmonic functions is set equal to one, the intersecting configuration should reduce to one of the D -brane solutions discussed in the introduction. The explicit form of the RR gauge fields is most easily given by using a formulation where the magnetic configurations are described by magnetic (dual) potentials. This leads us to consider the Lagrangian

$$\mathcal{L} = \sqrt{-g} \left\{ e^{-2\phi} \left[R - 4(\partial\phi)^2 \right] + \frac{(-)^{p+r+1}}{2(p+r+2)!} F_{(p+r+2)}^2 + \frac{(-)^{p+s+1}}{2(p+s+2)!} F_{(p+s+2)}^2 \right\}, \quad (17)$$

where it is understood that in the field equations one imposes the constraint that $F_{(8-p)}$ is the dual of $F_{(p+2)}$. In particular, $F_{(5)}$ is selfdual. Pseudo-Lagrangians of this form have been discussed in [24]. It is also understood that the two kinetic terms for the gauge fields become identical if $r = s$.

We next distinguish three different cases:

Case (1) Both harmonic functions depend on the overall transverse directions. The RR gauge fields are given by

$$F_{0\dots p1\dots ri}^{(1)} = \partial_i H_{p+r}^{-1}, \quad F_{0\dots p1\dots si}^{(2)} = \partial_i H_{p+s}^{-1}. \quad (18)$$

Case (2) The function H_{p+r} depends on the overall transverse directions whereas H_{p+s} depends on the relative transverse directions. The RR gauge fields are given by

$$F_{0\dots p1\dots ri}^{(1)} = H_{p+s}^\alpha \partial_i H_{p+r}^{-1}, \quad F_{0\dots p1\dots sr}^{(2)} = \partial_r H_{p+s}^{-1}. \quad (19)$$

Case (3) Both harmonic functions depend on the relative transverse directions. The RR gauge fields are given by

$$F_{0\dots p1\dots rs}^{(1)} = H_{p+s}^\alpha \partial_s H_{p+r}^{-1}, \quad F_{0\dots p1\dots sr}^{(2)} = H_{p+r}^\beta \partial_r H_{p+s}^{-1}. \quad (20)$$

As we will see below, one can also consider more general cases which are compositions of these three different cases. For simplicity we will often, if not necessary, omit the expressions for the RR fields⁶. Note that the α in case (2) and the α, β in case (3) are arbitrary (real) parameters that cannot be fixed by the Bianchi identities. They can be determined by the T -duality. Alternatively, we will determine them below by the equations of motion.

Sofar, we have only applied T -duality to generate the general Ansatz (13), (14), (18-20) for intersecting D -brane configurations. Our next task is to determine which of these configurations corresponds to a (supersymmetric) solution of the Lagrangian (17). Substituting our Ansatz into the vector field and dilaton equation⁷, we see that *case (1) can only be a solution for $n = 2$, case (2) for $n = 2$ and $\alpha = 0$ while case (3) requires that $n = 4$ and $\alpha = \beta = 1$* . Furthermore, it turns out that the cases (1) and (2) can naturally be combined into a more general configuration where H_{p+r} only depends on the overall transverse directions, as before, but where H_{p+s} is given by the sum of two harmonics $H_{p+s}^{(1)}, H_{p+s}^{(2)}$, which depend on the overall and relative transverse directions, respectively, i.e.

$$H_{p+s}(x^i, x^r) = H_{p+s}^{(1)}(x^i) + H_{p+s}^{(2)}(x^r). \quad (21)$$

We will now investigate the supersymmetry of these solutions. It has been shown in [2] that only those interesections can be supersymmetric (1/4 of the supersymmetry is unbroken) that satisfy the condition that $r + s = 0 \bmod 4$, i.e. $n = 2$ or 4. In our language this goes as follows. For a single D -brane the supersymmetry condition follows from $\delta\lambda = 0$, where λ is the spinor in the IIA/IIB supergravity multiplet, and $\delta\lambda$ (in the string frame) is given by

$$\delta\lambda = \gamma^\mu (\partial_\mu \phi) \epsilon + \frac{1}{4} \frac{3-p}{(p+2)!} e^\phi F_{\mu_1 \dots \mu_{p+2}} \gamma^{\mu_1 \dots \mu_{p+2}} \epsilon'_{(p)} = 0, \quad (22)$$

⁶Note that the RR gauge fields are contributing to the Wess-Zumino part of the world volume actions. As for the fundamental string the world volume actions provide the source terms of the classical solution.

⁷The case that only intersecting 3-Branes are involved is special since for this case the dilaton equation is trivially satisfied. By applying T -duality one can relate this case to the other cases and show that the same restrictions as given below apply.

where $\epsilon'_{(p)} = \epsilon$ for $p = 0, 4, 8$; $\epsilon'_{(p)} = \gamma_{11}\epsilon$ for $p = 2, 6$; $\epsilon'_{(p)} = i\epsilon$ for $p = -1, 7$ and $\epsilon'_{(p)} = i\epsilon^*$ for $p = 1, 5$ ⁸. Substituting the D -Brane solution into the above equation leads to the condition

$$\epsilon + \gamma_{01\dots p} \epsilon'_{(p)} = 0, \quad (23)$$

i.e. half of the supersymmetry is broken.

Now consider the intersection of a $(p+r)$ -Brane with a $(p+s)$ -Brane. Then the two supersymmetry conditions corresponding to the $(p+r)$ -Brane and $(p+s)$ -Brane are given by

$$\begin{aligned} \epsilon + \gamma_{01\dots p+r} \epsilon'_{(p+r)} &= 0, \\ \epsilon + \gamma_{01\dots p+s} \epsilon'_{(p+s)} &= 0, \end{aligned} \quad (24)$$

respectively. Combining the two supersymmetry conditions we get

$$\epsilon'_{(p+r)} = (-)^{\frac{1}{2}r(r+1)} \gamma_{r+s} \epsilon'_{(p+s)}. \quad (25)$$

We now distinguish four cases in which the two spinors in the above equation are given by (ϵ, ϵ) , $(\epsilon, \gamma_{11}\epsilon)$, $(i\epsilon, i\epsilon)$ or $(i\epsilon, i\epsilon^*)$, respectively. All four cases lead to the consistency condition that $\gamma_{r+s}^2 = 1$, or

$$n = 2 \quad \text{or} \quad 4, \quad (26)$$

thereby reproducing the condition of [2].

We next extend this analysis and consider the Killing spinor equation that follows from $\delta\lambda = 0$ for the case that we substitute the complete intersecting configuration and not only the separate D -Brane configurations. In the string-frame we obtain the following equation from $\delta\lambda = 0$:

$$\begin{aligned} \gamma^\mu (\partial_\mu \phi) \epsilon &+ \frac{1}{4} (3 - p - r) e^\phi F_{0\dots p+r, \mu}^{(1)} \gamma^{0\dots p+r, \mu} \epsilon'_{(p+r)} + \\ &+ \frac{1}{4} (3 - p - s) e^\phi F_{0\dots p+s, \mu}^{(2)} \gamma^{0\dots p+s, \mu} \epsilon'_{(p+s)} = 0. \end{aligned} \quad (27)$$

Substituting the explicit form of the general intersecting configuration (13), (14), (18-20) into the above Killing spinor equation leads, for case (1) to $n = 2$, for case (2) to $n = 2, \alpha = 0$ and for case (3) to $n = 4, \alpha = \beta = 1$. This nicely agrees with our earlier finding that only these configurations can be solutions to the equations of motion. It is not clear to us what the role is of the intersecting D -Branes with $n = 1, 3$, and whether they can be represented as some kind of solution.

⁸For $p = 3$ the supersymmetry condition does not follow from $\delta\lambda = 0$ (there is no dilaton and the 4-form gauge field is absent in $\delta\lambda$). For that case one has to consider the supersymmetry rule of the gravitino.

Finally, to construct solutions with more than two D -Branes, one can follow the same procedure. We take the configuration (13) for two intersecting D -Branes with $n = 2, 4$ and for the case that both Branes belong to the IIB theory and embed this solution into a Nine-Brane. Next, one should apply T -duality in the different possible directions to obtain the class of configurations describing three intersecting D -Branes. This process can be repeated. It would be interesting to determine which of these configurations can be (supersymmetric) solutions to the equations of motion.

3 Elementary D -Branes in 6 dimensions

In this section we describe the compactification of 2 intersecting D -Branes. In 6 dimensions we are interested in a basic set of non-intersecting D -Branes, which means that we have to compactify over the relative transversal space. Since the internal space has 4 dimensions we have to consider the class $n = 2$. From the table we conclude that there are 3 families given by the 3 columns in the $n = 2$ class, e.g. for a common 0-Brane ($p = 0$), we have $0\parallel 4$, $1\perp 3$ and $2\perp 2$. Since we are compactifying over a $K3$ we have to discard the second family. There is no place for a string inside the $K3$ (no 1-cycles). Thus we are left with the other two intersections. Next, we have to discuss how to put these objects into the $K3$. The $0\parallel 4$ object is clear, the 0-Brane is unchanged and the 4-Brane wraps around the whole $K3$. As a result we get one object defined by two charges. The two membranes intersecting over a point ($2\perp 2$) have two 2-cycles that have no common point. Obviously the two membranes have to wrap around two of the 22 2-cycles. Naively one would think that this gives us 22 possibilities where every possibility is defined by two charges. But instead, we have to take into account that the 2-cycles in the $K3$ pick up only the self- or antiself-dual part of the 2 membranes. Hence every possibility is related only to one charge. In total, the 6d solution is defined by 24 charges, that have to form a $O(4, 20)$ vector.

Therefore, a natural ansatz for the Type IIA D -0-Brane is given by

$$ds_{IIA}^2 = e^{2U} dt^2 - e^{-2U} ds_5^2, \quad e^{-4U} = e^{4\phi} = (|\vec{\chi}^R|^2 - |\vec{\chi}^L|^2), \quad (28)$$

where $\vec{\chi}^{R/L}$ is a harmonic vector in the $O(4, 20)$ space. To clarify this formula we consider the case $0\parallel 4$, where we have only two harmonic functions

$$\begin{pmatrix} \chi^R \\ \chi^L \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} H_0 + H_4 \\ H_0 - H_4 \end{pmatrix}, \quad (29)$$

where H_0 and H_4 are related to the two intersecting D -Branes in 10 dimensions with the metric given by

$$ds^2 = \frac{1}{\sqrt{H_0 H_4}} dt^2 - \sqrt{H_0 H_4} ds_5^2 - \sqrt{\frac{H_0}{H_4}} ds_4^2. \quad (30)$$

Wrapping the 4 relative transversal space around the $K3$ yields the following 6-dimensional metric

$$ds^2 = \frac{1}{\sqrt{H_0 H_4}} dt^2 - \sqrt{H_0 H_4} ds_5^2, \quad (31)$$

which is a special case of (28) and for $H_0 = H_4$ it coincides with (7).

As in 10 dimensions the electric gauge fields are proportional to the inverse power of a harmonic function and an $O(4, 20)$ covariant ansatz is (including the 16 left moving modes)

$$\vec{A}_0 = e^{4U} \begin{pmatrix} \vec{\chi}^R \\ \vec{\chi}^L \end{pmatrix}. \quad (32)$$

Also, there are scalar fields which are given by the matrix \mathcal{M} that parameterizes the $O(4, 20)$ space. To find this matrix and to proof that this is really a solution of the low energy effective action we have to compare the D -Brane solution with known solutions on the heterotic side (see below).

To find the dyonic D -1-Brane we have to compactify the intersections $1||5, 3\perp 3$ of the class $n = 2$. Again the internal structure of the $K3$ yields a result in 6 dimensions which has an $O(4, 20)$ structure

$$ds_{IIB}^2 = e^{2U} ds_2^2 - e^{-2U} ds_4^2, \quad e^{2\phi} = 1, \quad \vec{B}_{01} = \vec{A}_0, \quad (33)$$

where the other components are defined via the (anti)self-duality condition: $\vec{H} = (\mathcal{M} \cdot \mathcal{L}^* H)$ with the matrix \mathcal{L} defining the metric in the $O(4, 20)$ space (\mathcal{M} is the same form heterotic, IIA and IIB).

The magnetic 2-Branes can be obtained by reducing the $2||6$ and $4\perp 4$ solutions. The D -2-Brane in six dimensions has the form

$$ds_{IIA}^2 = e^{2U} ds_3^2 - e^{-2U} ds_3^2, \quad e^{2\phi} = e^{2U}, \quad \vec{F}_{ij} = \sqrt{2} \epsilon_{ijm} \partial_m \vec{\chi}. \quad (34)$$

The higher Branes are not asymptotically flat. For instance the Dirichlet 3-Branes in six dimensions are obtained from the reduction of the $3||7$ and $5\perp 5$ solutions. The 4-Brane is related to a cosmological constant and, finally, the 6d space time can be interpreted as the IIB 5-Brane. Like in 10 dimensions we can relate all these solution directly by T-duality.

So far, we have discussed the six-dimensional D -Branes as compactifications of two intersecting D -Branes in 10 dimensions. The $O(4, 20)$ structure was determined by the structure of the $K3$. To determine the scalar field matrix \mathcal{M} we have to look for the heterotic analogue. Starting from the Type IIA solutions we will find two heterotic solutions, a pure magnetic (by mapping the D -2-Brane) and one pure electric (by mapping the D -0-Brane). By using the string/string duality transformation we find for the pure magnetic solution the compactified (magnetic) chiral null model [25]

$$ds_H^2 = ds_3^2 - e^{-4U} dx^i dx^i, \quad e^{-4U} = e^{4\phi}$$

$$\mathcal{M} = \mathbf{1} + 2e^{4U} \begin{pmatrix} \chi_\alpha^L \chi_\beta^L & \chi_\alpha^L \chi_\beta^R \\ \chi_\alpha^R \chi_\beta^L & \xi \chi_\alpha^R \chi_\beta^R \end{pmatrix}, \quad \begin{pmatrix} \vec{F}_{ij}^L \\ \vec{F}_{ij}^R \end{pmatrix} = \sqrt{2} \epsilon_{ijm} \partial_m \begin{pmatrix} \vec{\chi}^L \\ \vec{\chi}^R \end{pmatrix}, \quad (35)$$

where $\xi = \frac{|\vec{\chi}^L|^2}{|\vec{\chi}^R|^2}$ and $i = 1, 2, 3$. Uplifted to 10 dimensions (via T_4) one finds

$$ds_H^2 = ds_3^2 - e^{-4U} dx^i dx^i (dx^\alpha + A_i^{(1)\alpha} dx^i) G_{\alpha\beta} (dx^\beta + A_i^{(1)\beta} dx^i) , \quad (36)$$

$$G_{\alpha\beta} = \delta_{\alpha\beta} - \frac{\chi_{(\alpha}^L \chi_{\beta)}^R}{|\vec{\chi}^R|^2 + (\vec{\chi}^R \vec{\chi}^L)} \quad , \quad e^{-2\phi} = e^{2U} \frac{1}{\sqrt{\det G}} ,$$

where x^α are the 4 isometry directions. The 10d antisymmetric tensor components are given by

$$B_{\alpha\beta} = \frac{-2\chi_{[\alpha}^L \chi_{\beta]}^R}{|\vec{\chi}^R|^2 + (\vec{\chi}^R \vec{\chi}^L)} \quad , \quad B_{\alpha\mu} = A_{\mu\alpha}^{(2)} + B_{\alpha\beta} A_\mu^{(1)\beta} , \quad (37)$$

where $A^{(1/2)}$ are the potentials related to the original field strength by (without the additional left moving part)

$$\begin{pmatrix} \vec{F}^{(1)} \\ -\vec{F}^{(2)} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \vec{F}^R \\ \vec{F}^L \end{pmatrix} . \quad (38)$$

The corresponding result for the pure electric case coincides with the solution in [26] which is an uplifted version of the solution found in [27]. This relation to known magnetic and electric solutions proofs that our D -Brane solutions are solutions of the Type IIA/B effective actions.

On the heterotic side we can interpret the solution as intersections of known basic p -Branes. To make this more transparent we take only 2 charges and go into the “(1)/(2)” basis. Then the 10d metric and dilaton (36) can be written as a magnetic chiral null model

$$ds_H^2 = ds_6^2 - H_s \tilde{H}_s dx^i dx^i - \frac{H_s}{\tilde{H}_s} (dz + A_i^{(1)} dx^i)^2 \quad , \quad e^{2\phi} = H_s . \quad (39)$$

For $\tilde{H}_s = 1$ and an appropriate torsion we get the (solitonic) 5-Brane and in the other case ($H_s = 1$) its T -dual, a Taub-NUT soliton. The 24 different charges in 6 dimensions are related to the different possibilities to choose the isometry direction z and to the possibility to give the 10-dimensional solution additional charges with respect to the left moving sector.

Similarly, if we transform the electric 0-Brane to the heterotic side we find a solution which is in 10 dimensions an intersection of a fundamental string (with the harmonic H_f) and its T -dual, the gravitational wave background (with the harmonic \tilde{H}_f)

$$ds^2 = \frac{1}{H_f} (dv du - \tilde{H}_f du^2) - ds_8^2 \quad , \quad e^{-2\phi} = H_f , \quad (40)$$

which is the electric chiral null model of [10].

4 Comments

We have shown that the elementary D -Brane solutions in six dimensions can be interpreted as the intersection of two D -Branes in ten dimensions. Furthermore we have shown

that the corresponding heterotic solutions can be viewed as the intersection of a string (or 5-Brane) with its T -dual. It is natural to extend this analysis and to consider the intersection of more than two solutions in ten dimensions which could involve both D -Branes and NS/NS solutions. Of special interest are those solutions which have vanishing scalars upon identification of the different harmonic functions involved.

As an example we consider the five-dimensional Reissner-Nordstrom black hole solution which was considered in [28] to give a microscopic derivation of the entropy in terms of counting D -Brane states. This solution has the metric

$$ds^2 = H^{-2}dt^2 - Hds_4^2, \quad (41)$$

where H is a harmonic function. A ten-dimensional origin of this solution has been discussed in [13]. Instead, here we will discuss its interpretation in terms of six-dimensional solutions. Given the powers of the harmonic function, it is clear that we should consider the intersection of a six-dimensional D -Brane with a NS/NS string or its T -dual. Requiring that we want to end up with a constant dilaton in five dimensions restricts ourselves to consider either the intersection of a 0-Brane with a fundamental string (cancelling dilatons) or a 1-Brane with the T -dual of a fundamental string (with each vanishing dilaton). Thus, in the first case the 6-dimensional intersecting solution is $1_f \times 0_D$ and is given by

$$ds^2 = \frac{1}{H_f H_0} dt^2 - \frac{H_0}{H_f} ds_1^2 - H_0 ds_5^2, \quad e^{-2\phi} = \frac{H_f}{H_0}, \quad (42)$$

whereas in the second case we have the intersecting solution $\tilde{1}_f \times 1_D$

$$ds^2 = \frac{1}{\tilde{H}_1} (dudv - \tilde{H}_f du^2) - \tilde{H}_1 ds_5^2, \quad e^{-2\phi} \sim 1. \quad (43)$$

Identifying the harmonic functions we obtain after compactification over ds_1 or u , respectively, the metric (41) with a vanishing dilaton and constant compactification radii. Looking on the 10-dimensional origin we find that this solution is given by 3 intersecting Branes, Type IIA for the first case and Type IIB for the second case. The Type IIB intersection has been discussed in [13].

As a second example for a solution that is interesting in the context of entropy calculations we discuss the 4-dimensional Reissner-Nordstrom solution (or their generalizations to more than one harmonic function). Again, for this solution there exists a limit obtained by identification of harmonic functions for which all scalars disappear. It is natural therefore to consider it as an elementary 4-dimensional D -Brane. The metric is given by

$$ds^2 = H^{-2}dt^2 - H^2 ds_3^2. \quad (44)$$

It has a natural interpretation in terms of intersecting D -Branes in 6 or 10 dimensions. Since the powers of the harmonic function in front of the world volume (time) and the transversal space is the same we can express this solution directly by D -Branes, we do not need the NS/NS-Branes. As in the case before we have two possibilities. The first

one with cancelling dilaton contributions is given by the intersection of a 0- and 2-Brane in 6 dimensions

$$ds^2 = \frac{1}{H_0 H_2} dt^2 - \frac{H_0}{H_2} ds_2^2 - H_0 H_2 ds_3^2 \quad , \quad e^{-2\phi} = \frac{H_2}{H_0} \quad , \quad (45)$$

while the second possibility with each vanishing dilaton is the intersection of two Dirichlet 1-Branes

$$ds^2 = \frac{1}{H_1 \tilde{H}_1} dt^2 - \frac{H_1}{\tilde{H}_1} ds_1^2 - \frac{\tilde{H}_1}{H_1} ds_1^2 - H_1 \tilde{H}_1 ds_3^2 \quad , \quad e^{-2\phi} \sim 1 \quad . \quad (46)$$

In analogy with the case of the 5-dimensional Reissner–Nordstrom black hole we find that upon identification of the harmonic functions (H_0 with H_2 or H_1 with \tilde{H}_1 , respectively) the compactification over the relative transversal space does not yield additional scalars and is given by (44). Since every 6-dimensional D -Brane has an interpretation as an intersection of two 10-dimensional D -Branes we obtain for this case in 10 dimensions an intersection of 4 D -Branes. Some examples of such an interpretation have been given in [17].

In order to find a non-vanishing area of the horizon (non-vanishing Bekenstein–Hawking entropy) it is not necessary to identify the harmonics. The crucial property is, that all scalar fields stay finite on the horizon ($r = 0$). Allowing for any Brane an independent harmonic function which means an independent charge yields the general case.

Our classification in terms of intersecting D -Branes has a natural interpretation in terms of the black hole solutions with dilaton coupling parameter $a = \sqrt{4/n - 1}$ [14]⁹:

- 1) $a = \sqrt{3}$: 10d elementary D -Brane;
- 2) $a = 1$: 6d elementary D -Brane (or 2 intersecting 10d D -Branes);
- 3) $a = 1/\sqrt{3}$: intersection of a 6d D -Brane with a NS/NS Brane (or 3 intersecting 10d Branes);
- 4) $a = 0$: 4d elementary D -Brane (or 2 intersecting 6d D -Branes or 4 intersecting 10d D -Branes).

So far we have only considered D -Branes. It is natural to also include anti- D -Branes which carry the opposite charge. In the case that both charges differ only in sign one obtains massless black holes [29]. This is consistent with the picture that the massless case in 4 dimensions corresponds to a composition of two or four $a = \sqrt{3}$ black holes [30].

Finally, we comment on the number of independent harmonic function in our intersecting solutions. Our basic set of D -Branes in 6 dimensions is given by 2 intersecting D -Branes in 10 dimensions and thus contains 2 harmonic functions. The 24 different charges are related to different radii in the $K3$. On the heterotic side (35) the solution can be naturally extended to more independent harmonic functions. At least for the left

⁹An interpretation of these solutions in terms of intersecting M -Branes has been discussed in [14, 17].

moving sector we can assume that the harmonic functions are completely independent yielding 1+4 functions in 6 dimensions (neglecting the 16 additional left moving modes). On the electric side this case corresponds in 10 dimensions to the chiral null model that contains more than two independent harmonic functions (the ω_n functions in [29]). These additional harmonic functions are related to momentum modes in the internal directions. In analogy, the magnetic chiral null model can be described by the same number of independent harmonic functions. It would be interesting to see what the role of the additional harmonic functions are in the D -Brane picture.

Note added: Soon after the appearance of this paper there appeared a preprint [31] which has some overlap with the present work.

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$(p+r)*(p+s)$ Intersecting D-branes

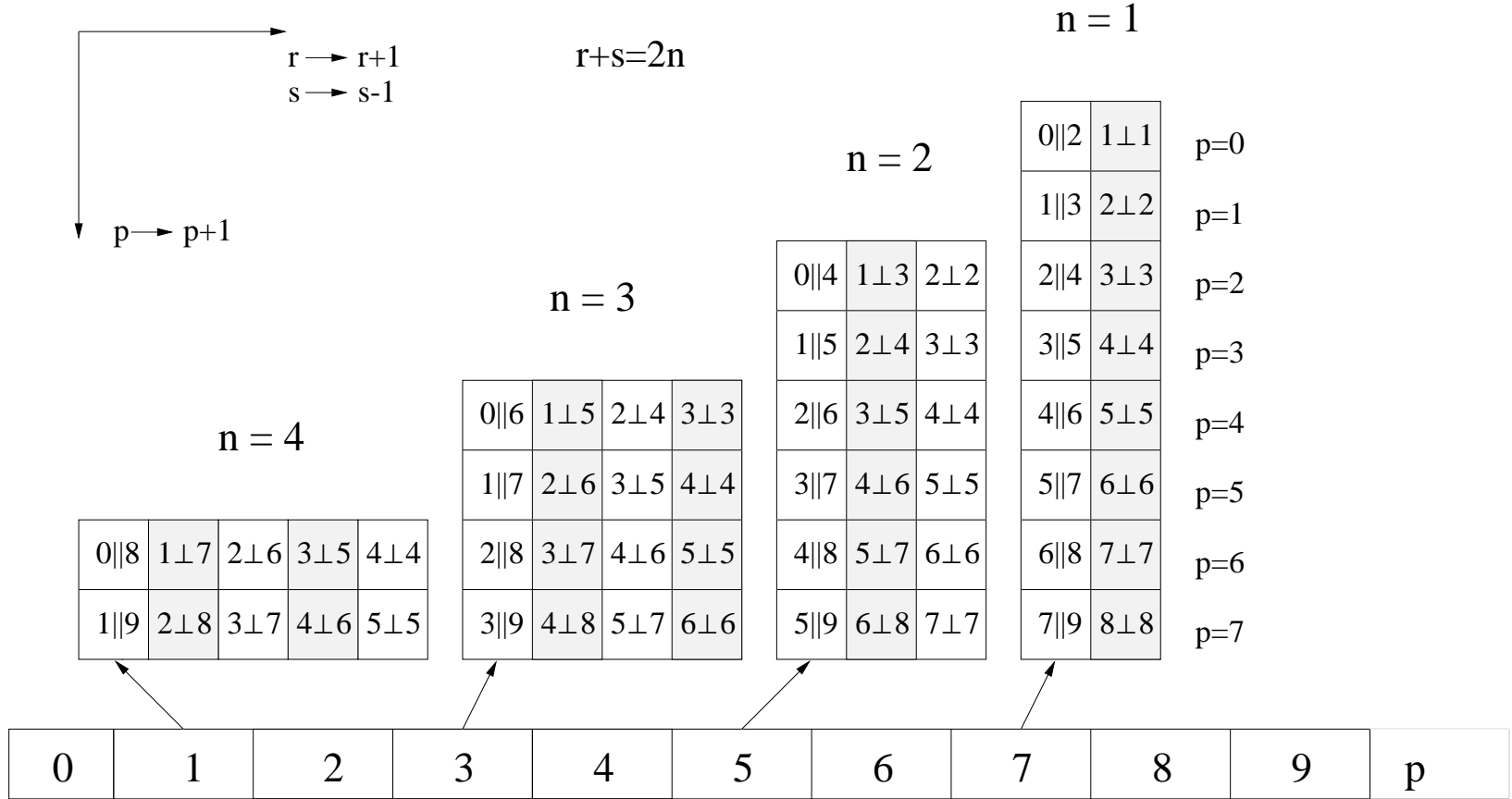


Table The table contains all possible intersecting configurations of two D-Branes in ten dimensions. The notation is explained in section 2. Note that each class starts at the top with $p = 0$ intersection and that p is constant inside the horizontal line of a given class.